

Susie: Hey Bob, want to see my cool new boots?

Bob: Sure, but didn't you just get new boots?

Susie: My boots last about a year and a half. And they only cost me about \$40. Aren't your boots really expensive?

Bob: They cost me about \$220, but I've had them for 9 years so far. I bet you have paid more for boots than I have.



Is Bob right? How do you know?

What price would Susie's boots have to be for her to spend the same amount as Bob? What assumptions did you have to make about their boots to solve the problem?

Can you create graph to represent the costs of their boots over time (assume Bob's boots need to be replaced after 12 years).

MATH STANDARDS ALIGNMENT

Grade 7: Ratios & Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.

CCSS.MATH.CONTENT.8.F.B.5

Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

CCSS.MATH.CONTENT.HSF.IF.C.7.B

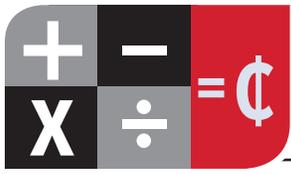
Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.

Personal Finance Big Ideas:

Inflation, What is Money, Time Value of Money



METHOD 1: UNIT PRICING PER YEAR

I notice after reading the problem that:

Bob paid \$220 for his boots.

Bob has worn his boots for 9 years already.

Susie paid \$40 for her boots.

Susie wears each pair of boots for 1.5 years.

I wonder if I can compare the money they spent on boots if I think of a "per year" cost. First I thought about what Bob spent.

\$220 divided by 9 years comes out to \$24.44 per year

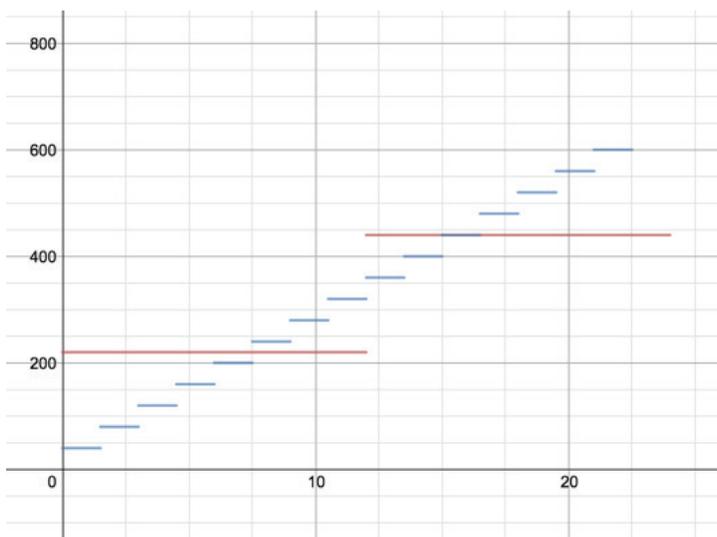
Next I thought about what Susie spent:

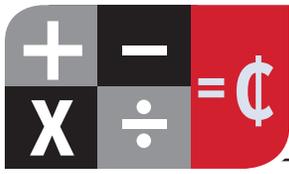
\$60 divided by 1.5 years comes out to \$26.67 per year

Bob is right that he spends less money on boots than Susie.

I wanted to think about how much Susie would have to spend for them to spend the same amount as Bob. For them to spend the same amount, Susie would have to pay Bob's unit price -- \$24.44 per year. But she buys boots every 1.5 years, so to find the cost of her boots, I have to add $\frac{1}{2}$ of \$24.44 to get the cost per pair of boots. $\frac{1}{2}$ of \$24.44 is \$12.22, so the cost for a pair of boots is $\$24.44 + \$12.22 = \$36.66$ if they were to spend the same amount on boots in 9 years.

To represent their spending as graphs, I realized that each of their spending was representing by a step function. The y-axis of my graph is the dollars spent on boots and the x-axis is number of years. The red lines in this graph are Bob's spending, he spends \$220 every 12 years. The blue lines are Susie's spending, she spends \$40 every 1.5 years:





METHOD 2: UNIT PRICING PER NINE YEARS

I notice after reading the problem that:

Bob paid \$220 for his boots.

Bob has worn his boots for 9 years already.

Susie paid \$40 for her boots.

Susie wears each pair of boots for 1.5 years.

I wonder if I can compare the money they spend on boots if I think about how much they each spend over a 9 year period. First I thought about what Bob spends:

\$220 for 9 years

Next I thought about what Susie spends. I used a ratio table to help me figure it out what Susie would have spent in 9 years.

Years	1.5	3	6	9
Dollars	40	80	160	240

That means that in 9 years, Susie will have spent \$240 on boots in years.

Bob is right that he has spent less money on boots than Susie has so far.

Let's assume Bob's boots lasts only 9 years and that we want Susie and Bob to have boots for all of the 9 years. Susie will have to buy boots 6 times to get through 9 years. For that to cost her \$220, we can divide \$220 by 6. That is \$36.67. So if she pays \$36.67 for each pair of boots, she will have spent the same amount as Bob over a nine year period.

When I graphed this I realized it was not linear. There was a set cost for a certain amount of years for each pair of boots, then it went up a set amount after that time was up and lasted for the same time. So it's what's called a step function, because it looks like steps. Since Bob's boots last longer and are more expensive, his steps are more vertical and horizontal for longer. Since Susie's boots are less expensive and she replaces them more often, her steps are shorter and increase vertically more often, but less distance than Bob's. For this graph, the x-axis is years and the y-axis is the cost in dollars. The red function is the cost of Bob's boots and the blue function is Susie's. You can see that for the second pair of Bob's boots there is a point around 15 years where the costs are the same for a 1.5 years for both boot investments, then Susie's continues to rise while Bob's remains constant.

