You are the director of a brand-new movie theater that is struggling to attract customers to its shows. You decide to put on a “Bring The Whole Gang” special sale: for every 2 tickets sold at full price, you will offer 3 tickets at a discounted price. You tell your bookkeeper to set the discounted price at whatever she wants, based on the full ticket price of $13.00 per person.

At the end of the night, your bookkeeper tells you that they sold 250 tickets -- a sellout crowd! As planned, 40% of the tickets sold for full price and 60% for the discounted price. “We made $2050 in gross ticket sales! That’s almost $1000 more than our previous top selling night.”

You ask your bookkeeper, “So was it a big discount or not? It got a lot more people to come, so it was probably a big discount. On the other hand, we still made a lot more money, so maybe it wasn’t that big?”

“Actually, you have more than enough information to figure that out!” she replies. How much did she charge for the discount?

Extra: Your bookkeeper explains that she polled the crowd and learned that for every fifty-cent increase in the price of the discounted ticket, 10 fewer people would have come. How can you use that information to set the discount price for your next “Bring the Whole Gang” night? To set your regular ticket price?

MATH STANDARDS ALIGNMENT
CCSS.MATH.CONTENT.6.RP.A.1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

CCSS.MATH.CONTENT.7.RP.A.2: Recognize and represent proportional relationships between quantities.

Personal Finance Big Ideas:
What is Money, Cost/Benefit Analysis, Opportunity Cost
METHOD 1: WORKING WITH PERCENTS
We noticed that 40% of the tickets sold were sold at full price, and 60% were sold at the discounted price. Because they told us 250 tickets were sold, we realized we could find out how many of each type of ticket was sold. 40% means that for every 100 tickets sold, 40 were full price.

We figured out 40% of 250 by thinking of groups of 100. 250 = 100 + 100 + 50. So each 100 represents 40 full price tickets, and out of the 50 (half of 100), there were 20 full price tickets sold.

40% of 250 = 40 + 40 + 20 = 100.

That means the other 150 tickets sold were discounted tickets.

The story said that full-price tickets cost $13.00. That means that they made $13 per ticket * 100 tickets = $1300 from just the full-price tickets.

They made $2050 all together. Since $1300 of that $2050 came from full-price tickets, the other $750 had to come from discounted tickets. $2050 - $1300 = $750.

We already figured out that 150 tickets were sold at the discounted price. Since we know the total amount made from discounted ticket sales, and the total number of tickets sold, we can divide to find the discounted price.

$750 ÷ 150 = $5.

The discounted tickets were $5 each. That’s a really big discount — more than 50% off, and more than half the tickets sold were at that price. We think the theater should lower their regular price and hopefully get more people to come each night.

METHOD 2: FINDING THE NUMBER OF GROUPS OF TICKETS SOLD
After reading the problem, our group realized that for every 5 tickets sold, 2 were regular price and 3 were discounted. We thought of the 250 tickets sold and wondered how many groups of 5 tickets that represented. 250 ÷ 5 = 50. So they sold 50 groups of tickets, with 2 at the regular price and 3 at the discounted price.

Then we thought about the total amount of money they made: $2050. If 50 groups bought tickets, how much did each group pay?

$2050 ÷ 50 = $41.

Each group paid $41 for all of their tickets. How much did the discounted people pay?

The regular tickets cost $13, so the two regular tickets cost $26 all together. That means the three discounted tickets cost $41 - $26 all together. $41 - $26 = $15.

The three discounted tickets cost $15 together, so they must each cost $5. $15 ÷ 3 = $5.
METHOD 3: MAKE A RATIO TABLE
After reading the problem our group realized that there was a ratio given in the problem and there was another ratio that we wanted to reach. We decided to use a ratio table as a way to go from our regular to discount ticket ratio of 2 to 3 to the ratio of total regular tickets sold to total discount tickets sold. We started with our 2:3 ratio and knew that as we multiplied the ratio by 2 each time, the total number of tickets had to equal 250. Once we multiplied 160 by 2 and went over our target number of 250, we stopped to think about what to do next.

<table>
<thead>
<tr>
<th>x2</th>
<th>x2</th>
<th>x2</th>
<th>x2</th>
<th>x2</th>
<th>x2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular tickets sold</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>Discount tickets sold</td>
<td>3</td>
<td>6</td>
<td>12</td>
<td>24</td>
<td>48</td>
</tr>
<tr>
<td>Total Tickets sold</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>40</td>
<td>80</td>
</tr>
</tbody>
</table>

We realized that after we had multiplied by 2 five times (which is the same as multiplying by 32, since 2 x 2 x 2 x 2 x 2 is 32), we were at 160 total tickets. That is 90 away from our goal of 250 tickets. We realized that for every 2 regular tickets sold, 3 discount tickets were sold for a total of 5 tickets. We divided our remaining 90 tickets we needed by 5 and got 18 more groups of tickets. Adding 18 to 32 gave us 50, so we decided to multiply our 2:3 ratio by 50 groups of tickets to see if we could reach our target number:

<table>
<thead>
<tr>
<th>x 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular tickets sold</td>
</tr>
<tr>
<td>Discount tickets sold</td>
</tr>
<tr>
<td>Total Tickets sold</td>
</tr>
</tbody>
</table>

It worked! We knew each regular ticket was $13, so 13 x 100 = $1300 in regular ticket profits. This subtracted from our total profit of $2050 gave us $750 in discount ticket profits. To find out the discounted ticket price, we divided the $750 in discount ticket profits by the 150 discount tickets sold and got $5 per discount ticket.

METHOD 5: I NOTICE, I WONDER™ AND ALGEBRA
After reading the problem, our group decided to write down everything we noticed.

- I am the manager of a new movie theater
- The theater is not selling enough tickets
- There is a discount called “Bring the Whole Gang”
- For every two regular price tickets, three discounted tickets will be sold
- Regular price tickets cost $13
- The bookkeeper gets to decide on the discount price
- There are 250 seats
- All 250 seats are sold
- 40% of the tickets sold are regular price
- 60% of the tickets sold are discounted
- The movie theater made $2050
- The ratio of regular to discounted tickets is 2 to 3
- Bookkeeper has two o’s, two k’s, and two e’s in a row
We wondered:

- Was it a popular movie?
- How many discounted tickets were sold?
- How many regular tickets were sold?
- How much was the discount?
- Was it a big discount?
- What are gross ticket sales? What is gross about the tickets?
- What would have happened if they hadn’t had the discount?
- What would happen if they lowered the regular ticket price?
- Why didn’t people want to come?

To solve the problem, we first had to figure out how many of each type of ticket were sold. The ratio of regular to discounted tickets is 2 to 3, so if \( r \) represents the number of regular tickets:

\[
\frac{2}{5} = \frac{r}{250}
\]

\[
\frac{2}{5} \times 250 = \left( \frac{r}{250} \right) \times 250
\]

\[
\frac{500}{5} = r
\]

\[
100 = r
\]

That means 100 regular-price tickets were sold. Regular price tickets are $13 each so $13 \times 100 student tickets = $1300. Subtracting that from the total $2050 made from the sale we got $2050 - $1300 = $750 made on discount tickets.

To find out the discounted ticket price, we can divided the $750 in discount ticket profits by the number of discount tickets sold. At first we thought we would have to solve another proportion but then we realized that since 250 tickets were sold and 100 were at regular price, the number of discounted tickets is 250 - 100 = 150. We divided $750 by 150 and got $5 per discount ticket.

**Extra:** The story said that for every fifty-cent increase in discounted ticket price, 10 fewer people would have come. That made us think of making a table with the different ticket prices, attendance, and ticket sales.

We started with the ticket price of $5.00 and 250 people coming, with 40% paying full price and 60% paying the discount. Then we increased the ticket price by $0.50 and decreased the total number of ticket sales by 10. We used a spreadsheet and had it calculate:

\[0.40 \times \text{ticket sales}\]

to find the number of full price tickets. We had it do:

\[0.60 \times \text{ticket sales}\]
to calculate the number of discounted tickets. Then we had it add
13 * number of full price tickets + sale price * number of discounted tickets
to find the total revenue. We looked to see if we could make the total revenue greater than $2050.

Table 1-1

<table>
<thead>
<tr>
<th>Discount Ticket Price</th>
<th>Number of People</th>
<th>Number Paying Full Price</th>
<th>Number Paying Discount</th>
<th>Total Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>250</td>
<td>100</td>
<td>150</td>
<td>2050</td>
</tr>
<tr>
<td>5.50</td>
<td>245</td>
<td>98</td>
<td>147</td>
<td>2082.5</td>
</tr>
<tr>
<td>6.00</td>
<td>235</td>
<td>94</td>
<td>141</td>
<td>2068</td>
</tr>
<tr>
<td>6.50</td>
<td>225</td>
<td>90</td>
<td>135</td>
<td>2047.5</td>
</tr>
<tr>
<td>7.00</td>
<td>215</td>
<td>86</td>
<td>129</td>
<td>2021</td>
</tr>
<tr>
<td>7.50</td>
<td>205</td>
<td>82</td>
<td>123</td>
<td>1988.5</td>
</tr>
<tr>
<td>8.00</td>
<td>195</td>
<td>78</td>
<td>117</td>
<td>1950</td>
</tr>
<tr>
<td>8.50</td>
<td>185</td>
<td>74</td>
<td>111</td>
<td>1905.5</td>
</tr>
<tr>
<td>9.00</td>
<td>175</td>
<td>70</td>
<td>105</td>
<td>1855</td>
</tr>
<tr>
<td>9.50</td>
<td>165</td>
<td>66</td>
<td>99</td>
<td>1798.5</td>
</tr>
<tr>
<td>10.00</td>
<td>155</td>
<td>62</td>
<td>93</td>
<td>1736</td>
</tr>
<tr>
<td>10.50</td>
<td>145</td>
<td>58</td>
<td>87</td>
<td>1667.5</td>
</tr>
</tbody>
</table>

The most revenue we could get is when the ticket prices were $5.50. The revenue went up at first and then started to come down again. The bookkeeper did a good job picking a ticket price that would make almost the most possible amount of money!

We also thought about the regular ticket price. If you charged $5.50 for the discounted ticket and $13 for the regular tickets, every 5 people that came would pay

\[ \$5.50 + \$5.50 + \$5.50 + \$13 + \$13 = \$42.50 \]

If they all split the costs evenly instead of some people paying a lot more, they would each pay:

\[ \$42.50 ÷ 5 = \$8.50 \]

So maybe if you just charged $8.50 per ticket, 245 people would come to the shows, and you would make $2082.50 at each show.