As I waited for the subway this morning I started keeping a tally of how people paid the fare. I counted 20 people before my train arrived. Twelve paid with tokens, six paid with transit cards, and two paid cash. I decided to assume that these proportions were typical of the people who ride the morning subway.

When I squeezed onto the train, I quickly counted 47 people sitting and 23 (including myself) standing in the train car.

Questions: Using my assumptions about how people paid for their morning ride, how many of the people in that train car paid with a token? How many paid with a transit card? How many paid cash?

Extra: Tokens cost $1.30 each. A cash fare is $2.00. A transit card costs $69 per month. I estimate that people who buy transit cards use them for 60 rides per month. How much did it cost the people in my train car to ride the subway this morning?

MATH STANDARDS ALIGNMENT:
Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
7. Look for and make use of structure

Personal Finance Big Ideas:
Cost/Benefit Analysis
METHOD 1: MAKE A TABLE

I made a table to think about how the riders pay for the subway and how many there were waiting:

<table>
<thead>
<tr>
<th>tokens</th>
<th>transit card</th>
<th>cash</th>
<th># riders</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>6</td>
<td>2</td>
<td>20</td>
</tr>
</tbody>
</table>

I notice that all those numbers are even and so I can compare them like this, too, since the problem says we should assume that the proportions are going to stay the same:

<table>
<thead>
<tr>
<th>tokens</th>
<th>transit card</th>
<th>cash</th>
<th># riders</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>6</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

Once in the subway car, there were 47 people sitting and 23 people standing for a total of 70 people. I add the 70 to my table to think about what the other numbers might be.

<table>
<thead>
<tr>
<th>tokens</th>
<th>transit card</th>
<th>cash</th>
<th># riders</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>6</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>7 • 6</td>
<td>7 • 3</td>
<td>7 • 1</td>
<td>7 • 10</td>
</tr>
<tr>
<td>42</td>
<td>21</td>
<td>7</td>
<td>70</td>
</tr>
</tbody>
</table>

For the total of 70 people riding the subway, there were 42 people with tokens, 21 people with transit cards and 7 paying with cash.

Extra: Here we multiply the number of people using each method by the cost for each method. To do this we must first determine the average cost of one trip for transit card users:

$69.00 / 60 rides = $1.15 per ride

We then use this information to calculate subtotals:

42 paid with tokens • $1.30/token = $54.60
21 used transit cards • $1.15 = $24.15
7 paid cash • $2.00 = $14.00

Finally, we sum the subgroups to find the total of $92.75
METHOD 2: DRAW A TALLY CHART AND USE PROPORTIONS

After reading the problem, I started by re-creating the tally chart. I think it would have looked something like this:

<table>
<thead>
<tr>
<th>tokens</th>
<th>transit cards</th>
<th>cash</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The problem said that these are the proportions so that would mean:

tokens : transit cards : cash
12 : 6 : 2

which is the same as this proportion:
6 : 3 : 1

The problem said that there were 47 + 23 people or a total of 70 people. My basic proportion accounts for 10 people at a time. If I use that proportion seven times then I get:
42 : 21 : 7

There were 42 people with tokens, 21 people with transit cards and 7 paying with cash for a total of 70 people riding the subway.

METHOD 3: USE PERCENTS

The problem states that the first group of people observed were:
12 paying with tokens, 6 using transit cards, and 2 paying with cash.

That’s a total of 20 people. I can think of this as two groups of ten and so one group of ten people would be:
6 out of 10 with tokens, 3 out of 10 using transit cards, 1 out of 10 paying cash

If I take that one more step so that I can think of percents, I have:
60 out of 100 (60%) with tokens, 30 out of 100 (30%) with cards, 1 out of 10 (10%) paying cash

I know that there are 47 people seated plus 23 people standing and so there are 70 people total.
60% of 70 is 42 people who pay with tokens.
30% of 70 is 21 people who pay with transit cards.
10% of 70 is 7 people who pay with cash.
METHOD 4: USE FRACTIONS AND A FRACTION BAR

I notice this information stated in the problem:

- 20 people were waiting for the subway train

\[
\frac{12}{20} \quad \text{or} \quad \frac{6}{10} \quad \text{paid with tokens}
\]

\[
\frac{6}{20} \quad \text{or} \quad \frac{3}{10} \quad \text{paid with transit cards}
\]

\[
\frac{2}{20} \quad \text{or} \quad \frac{1}{10} \quad \text{paid cash}
\]

I drew this fraction bar to show how the whole (10 people) would be divided:

[Diagram of fraction bar]

The light grey shows the 6 out of 10 token users. The dark grey shows the 3 out of 10 transit card users and the white shows 1 out of 10 cash users.

I notice that once everyone is in the subway car there are a total of 70 people (47 + 23). Now instead of my whole being 10, it is 70. That would mean that each of the fraction bar segments would represent 7 people. I would now have:

\[
\frac{6}{10} \quad \cdot \quad \frac{7}{7} = \frac{42}{70}
\]

\[
\frac{3}{10} \quad \cdot \quad \frac{7}{7} = \frac{21}{70}
\]

\[
\frac{1}{10} \quad \cdot \quad \frac{7}{7} = \frac{7}{70}
\]

Of the 70 people, there were 42 people with tokens, 21 people with transit cards and 7 paying with cash.

METHOD 5: SET UP A PROPORTION

I notice this information stated in the problem:

- 20 people were waiting for the subway train
- 12 out of 20 paid with tokens
- 6 out of 20 paid with transit cards
- 2 out of 20 paid cash
- 47 + 23 or 70 people were standing or sitting in the subway train
- the proportions noted while waiting are assumed to be the same proportions for the riders
I can set up this proportion to find the number of people who paid with tokens:

\[
\frac{12 \text{ tokens}}{20 \text{ people}} = \frac{? \text{ tokens}}{70 \text{ people}}
\]

\[
\frac{6 \text{ tokens}}{10 \text{ people}} = \frac{? \text{ tokens}}{70 \text{ people}}
\]

\[
\frac{42 \text{ tokens}}{20 \text{ people}} = \frac{? \text{ tokens}}{70 \text{ people}}
\]

\[
42 = ?
\]

42 of the riders paid with tokens.

Similarly, I can use proportions to find the number who paid with transit cards or cash:

\[
\frac{6 \text{ tokens}}{20 \text{ people}} = \frac{? \text{ tokens}}{70 \text{ people}}
\]

\[
\frac{3 \text{ tokens}}{10 \text{ people}} = \frac{? \text{ tokens}}{70 \text{ people}}
\]

\[
\frac{21 \text{ tokens}}{20 \text{ people}} = \frac{? \text{ tokens}}{70 \text{ people}}
\]

\[
21 = ?
\]

21 of the riders paid with transit cards and 7 paid with cash.