At 3:26 PM on Tuesday, December 30, 2003, Ben’s family was stuck in a steady stream of traffic waiting to go through the Lincoln Tunnel to drive into Manhattan. They couldn’t help but have a little fun with math while they waited. Ben observed that it took 8 seconds on average for each car to pay and get through the tollbooth. Seven tollbooths were open. Each vehicle paid $6.00 to enter the tunnel.

Question: How much money did the Transit Authority collect between 3:30 pm to 4:30 pm if the rates at which the cars pass through the tollbooths stayed the same for the entire hour?

Extra: In addition to the tollbooths that collect cash, there are EZPass lanes. For these, you drive slowly through the toll area without stopping and the cost of the toll is automatically deducted from your EZPass account. If you use EZPass, the cost of the toll is $4.00 during off-peak hours and $5.00 during peak hours. Afternoon peak hours are from 4:00 p.m. to 7:00 p.m. (You save money because the Transit Authority doesn’t have to pay anyone to collect the tolls).

If cars drive through each of the four EZPass lanes once every 5 seconds, how much money does the Transit Authority collect from the EZPass lanes during that same hour?

MATH STANDARDS ALIGNMENT:
Grade 3: Operations & Algebraic Thinking
- Represent and solve problems involving multiplication and division.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Grade 4: Operations & Algebraic Thinking
- Use the four operations with whole numbers to solve problems.

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Construct viable arguments and critique the reasoning of others.

Personal Finance Big Ideas: What is Money
METHOD 1: MAKE A TABLE

I made a table to think about how the riders pay for the subway and how many there were waiting:

<table>
<thead>
<tr>
<th>tokens</th>
<th>transit card</th>
<th>cash</th>
<th># riders</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>6</td>
<td>2</td>
<td>20</td>
</tr>
</tbody>
</table>

I notice that all those numbers are even and so I can compare them like this, too, since the problem says we should assume that the proportions are going to stay the same:

<table>
<thead>
<tr>
<th>tokens</th>
<th>transit card</th>
<th>cash</th>
<th># riders</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>6</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

Once in the subway car, there were 47 people sitting and 23 people standing for a total of 70 people. I add the 70 to my table to think about what the other numbers might be.

<table>
<thead>
<tr>
<th>tokens</th>
<th>transit card</th>
<th>cash</th>
<th># riders</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>6</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>70</td>
</tr>
</tbody>
</table>

For the total of 70 people riding the subway, there were 42 people with tokens, 21 people with transit cards and 7 paying with cash.

Extra: Here we multiply the number of people using each method by the cost for each method. To do this we must first determine the average cost of one trip for transit card users:

\[
\frac{69.00}{60 \text{ rides}} = \$1.15 \text{ per ride}
\]

We then use this information to calculate subtotals:

- 42 paid with tokens \( \times \$1.30/\text{token} = \$54.60 \)
- 21 used transit cards \( \times \$1.15 = \$24.15 \)
- 7 paid cash \( \times \$2.00 = \$14.00 \)

Finally, we sum the subgroups to find the total of $92.75
METHOD 2: DRAW A TALLY CHART AND USE PROPORTIONS
I was not sure how much each pair of socks was, so I thought I would use the guess and check strategy. I decided to start by seeing how much the total would be if each pair of socks was $5.

Checking $5 per pair:
I know that ten 5s are $50. So nine 5s are $45. That’s way too high!

I’ll try $3 per pair:
I know that ten 3s are $30. So nine 3s are $27. Still too high, but closer!

I’ll try $2 a pair.
I know that ten 2s are $20. So nine 2s are $18. Got it!!

Each pair of socks is $2.

METHOD 3: GUESS AND CHECK – SKIP COUNTING
After reading the problem, I started by re-creating the tally chart. I think it would have looked something like this:

<table>
<thead>
<tr>
<th>tokens</th>
<th>transit cards</th>
<th>cash</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The problem said that these are the proportions so that would mean:
- tokens : transit cards : cash
  - $12 : 6 : 2$

which is the same as this proportion:
- $6 : 3 : 1$

The problem said that there were 47 + 23 people or a total of 70 people. My basic proportion accounts for 10 people at a time. If I use that proportion seven times then I get:
- $42 : 21 : 7$

There were 42 people with tokens, 21 people with transit cards and 7 paying with cash for a total of 70 people riding the subway.

METHOD 3: USE PERCENTS
The problem states that the first group of people observed were:
- 12 paying with tokens, 6 using transit cards, and 2 paying with cash.

That’s a total of 20 people. I can think of this as two groups of ten and so one group of ten people would be:
- 6 out of 10 with tokens, 3 out of 10 using transit cards, 1 out of 10 paying cash

If I take that one more step so that I can think of percents, I have:
- 60 out of 100 (60%) with tokens, 30 out of 100 (30%) with cards, 1 out of 10 (10%) paying cash
I know that there are 47 people seated plus 23 people standing and so there are 70 people total.

- 60% of 70 is 42 people who pay with tokens.
- 30% of 70 is 21 people who pay with transit cards.
- 10% of 70 is 7 people who pay with cash.

**METHOD 4: USE FRACTIONS AND A FRACTION BAR**

I notice this information stated in the problem:

- 20 people were waiting for the subway train

\[
\begin{align*}
\frac{12}{20} & \text{ or } \frac{6}{10} \text{ paid with tokens} \\
\frac{6}{20} & \text{ or } \frac{3}{10} \text{ paid with transit cards} \\
\frac{2}{20} & \text{ or } \frac{1}{10} \text{ paid cash}
\end{align*}
\]

I drew this fraction bar to show how the whole (10 people) would be divided:

- The light grey shows the 6 out of 10 token users.
- The dark grey shows the 3 out of 10 transit card users.
- The white shows 1 out of 10 cash users.

I notice that once everyone is in the subway car there are a total of 70 people (47 + 23). Now instead of my whole being 10, it is 70. That would mean that each of the fraction bar segments would represent 7 people. I would now have:

\[
\begin{align*}
\frac{6}{10} \cdot \frac{7}{7} &= \frac{42}{70} \\
\frac{3}{10} \cdot \frac{7}{7} &= \frac{21}{70} \\
\frac{1}{10} \cdot \frac{7}{7} &= \frac{7}{70}
\end{align*}
\]

Of the 70 people, there were 42 people with tokens, 21 people with transit cards and 7 paying with cash.

**METHOD 5: SET UP A PROPORTION**

I notice this information stated in the problem:

- 20 people were waiting for the subway train
- 12 out of 20 paid with tokens
- 6 out of 20 paid with transit cards
- 2 out of 20 paid cash
- 47 + 23 or 70 people were standing or sitting in the subway train
- the proportions noted while waiting are assumed to be the same proportions for the riders
I can set up this proportion to find the number of people who paid with tokens:

\[
\begin{align*}
\frac{12 \text{ tokens}}{20 \text{ people}} &= \frac{? \text{ tokens}}{70 \text{ people}} \\
\frac{6 \text{ tokens}}{10 \text{ people}} &= \frac{? \text{ tokens}}{70 \text{ people}} \\
\frac{42 \text{ tokens}}{70 \text{ people}} &= \frac{? \text{ tokens}}{70 \text{ people}} \\
\frac{42 \text{ tokens}}{42} &= ?
\end{align*}
\]

42 of the riders paid with tokens.

Similarly, I can use proportions to find the number who paid with transit cards or cash:

\[
\begin{align*}
\frac{6 \text{ tokens}}{20 \text{ people}} &= \frac{? \text{ tokens}}{70 \text{ people}} \\
\frac{3 \text{ tokens}}{10 \text{ people}} &= \frac{? \text{ tokens}}{70 \text{ people}} \\
\frac{21 \text{ tokens}}{70 \text{ people}} &= \frac{? \text{ tokens}}{70 \text{ people}} \\
\frac{21 \text{ tokens}}{21} &= ?
\end{align*}
\]

21 of the riders paid with transit cards and 7 paid with cash.